Uitwerking tentamen mechanica 16-3-98.

1 a. $g = G\frac{M_{Io}}{r^2} = 6{,}67\cdot10^{-11}\cdot\frac{0{,}079\cdot10^{24}}{(1{,}8\cdot10^6)^2} = 1{,}63\ m/s^2$

6

b $F_g = F_{mpz} \Rightarrow G\frac{M_1 M_2}{R^2} = M_2\,\omega^2 . R \qquad \omega = \sqrt{\frac{G M_1}{R^3}} = 4{,}1\cdot10^{-5}\ rad/s$

6

c $N = F_{g_{Io}} - F_{g_{Jup}} + F_{mpz} = 1{,}626 - 6{,}67\cdot10^{-11}\frac{19\cdot10^{26}}{(419{,}8\cdot10^6)^2} + (4{,}1\cdot10^{-5})^2\cdot 419{,}8\cdot10^6 =$

6

$1{,}626 - 0{,}7191 + 0{,}7057 = 1{,}613$

$\Rightarrow N = 1{,}61\ N$

2 a W.vb. v.I : in y-richting: $m_1 V_1 \sin\theta = m_2 V_2 \sin\theta \Rightarrow m_1 V_1 = m_2 V_2$

in x-richting: $m_1 V_0 = (m_1 V_1 + m_2 V_2)\cos\theta \qquad V_1 = \frac{V_0}{2\cos\theta}$

6

$V_2 = \frac{V_0}{2\cos\theta}\ \frac{m_1}{m_2}$

b $\Delta T = \frac{1}{2}m_1 V_0^2 - \frac{1}{2}m_1 V_1^2 - \frac{1}{2}m_2 V_2^2 = \frac{1}{2}m_1 V_0^2 - \frac{1}{2}m_1\left(\frac{V_0}{2\cos\theta}\right)^2 - \frac{1}{2}m_2\left(\frac{m_1}{m_2}\frac{V_0}{2\cos\theta}\right)^2 =$

6 $\frac{1}{2}m_1 V_0^2\left(1 - \frac{m_1+m_2}{4m_2\cos^2\theta}\right)$

c $\Delta T = 0 \Rightarrow 1 - \frac{m_1+m_2}{4m_2\cos^2\theta} = 0 \Rightarrow \cos^2\theta = \frac{m_1+m_2}{4m_2} = \frac{m_1}{4m_2} + \frac{1}{4} \geq \frac{1}{4}$

6 $\Rightarrow 0 \leq \theta \leq \frac{\pi}{3}\ (=60^\circ)$

3 a $U = mgy = mg(h - s.\theta)$

6

b $T = \frac{1}{2}I\dot\theta^2 + \frac{1}{2}m\dot y^2 = \frac{1}{4}mR^2\dot\theta^2 + \frac{1}{2}ms^2\dot\theta^2 = \frac{1}{2}m\left(\frac{1}{2}R^2 + s^2\right)\dot\theta^2$

6

c $\frac{\partial L}{\partial\theta} - \frac{d}{dt}\frac{\partial L}{\partial\dot\theta} = m\left(\frac{1}{2}R^2+s^2\right)\ddot\theta - mgs = 0 \Rightarrow \ddot\theta = \frac{2gs}{R^2+2s^2}$

6

d energie behoud : $\frac{1}{2}m\left(\frac{1}{2}R^2+s^2\right)\dot\theta^2 + mg(h - s.\theta) = mgh$

6

als de schijf beneden is : $h = s.\theta \Rightarrow \dot\theta^2 = (\omega^2 =)\ \frac{4gh}{R^2+2s^2}$

$\dot y\ (y=0) = s.\dot\theta = 2s\sqrt{\frac{gh}{R^2+2s^2}}$

e $t = \frac{h}{\langle v_y\rangle} = \frac{h}{\frac{\dot y(y=0)}{2}} = \frac{h}{s\sqrt{\frac{gh}{R^2+2s^2}}} = \frac{1}{s}\sqrt{\frac{h(R^2+2s^2)}{g}}$

6

4. Voor de massieve cilinder geldt: $mgh = \frac{1}{2}mV_1^2 + \frac{1}{2}\cdot\frac{1}{2}mR^2\omega^2 = \frac{3}{4}mV^2 \Rightarrow gh = \frac{3}{4}V_1^2$

voor de holle cilinder geldt : $mgh = \frac{1}{2}mV_2^2 + \frac{1}{2}mR^2\omega^2 = mV^2 \Rightarrow gh = V_2^2$

12 $\Rightarrow V_1$ is altijd groter dan V_2 ; onafh. van m of R

5 Door de tol wordt een "koppel" uitgeoefend met een vector in de + x richting (L^*)

12 $\Rightarrow$ de optelsom $\vec L + \vec{L^*}$ levert een verdraaiing in de + x richting.